A Phase Based Image Comparison Technique

Gian Paolo Lorenzetto and Peter Kovesi
Department of Computer Science
University of Western Australia
Nedlands, W.A., 6907
gian@cs.uwa.edu.au
pk@cs.uwa.edu.au

Abstract

Two of the most popular image quality measures are the root mean square and signal-to-noise ratio. Unfortunately these measures are simple tallies of pixel difference and provide no information about the type of degradation present. Although simple, and consequently provide computational benefits, the RMS and SNR measures cannot meaningfully be applied to images containing text or binary images. Pixel tally measures are also unable to measure perceptual *distortion. Take the case of two identical images, one translated one place to the right. These images still appear similar, but an RMS type error will return a large difference.*

A new approach, based in the frequency domain, to measuring the quality of an image is presented. In particular a frequency based approach allows for the measurement of distortion regardless of translation. Results for the new measure are also presented.

1. Introduction

Given that there are few tasks as subjective as that of a human viewer comparing two images, how can image quality be quantified? Two of the most popular error measures are the *root mean square* (RMS) error, and the *signal-to-noise ratio* (SNR). Both these measures are of little use in determining image quality since they are merely tallies of pixel difference. Two images can have disparate pixel values at corresponding points yet still appear identical. Consider two images differing only by a translation. These images have no correspondence in terms of grey values, yet still appear the same.

However, although inherently flawed, there are two reasons the RMS and SNR errors are still in use. First, the simplicity of the RMS error means it has a computational advantage over more complicated approaches, and second it provides a theoretically sound, statistical measure of the mean difference in pixel intensity. The RMS is not useful however, when applied to binary images, images involving text, or translated images.

Another problem of the RMS error is that it fails to provide an indication as to the type of degradation in an image. It is important to understand the difference between two images before making a judgment as to the quality of the image. A measure of an image's "quality" should be invariant to the translation, whereas the RMS error is unable to distinguish between translation and degradation. Moreover, a true measure of image quality is one that can separate the many different aspects of distortion between two images. These aspects include image translation, rotation, compression, and enhancement.

There are many existing techniques for measuring image quality, but like the RMS, all have particular flaws, or limited domains [3]. To date there is no single measure of image quality, due to several factors. The first is that the extreme complexity of the human visual system suggests that a simple method, or one giving a single number result are not useful. Secondly, validation of an image quality measure can only be performed by psychophysical tests, involving human viewers ranking images, and comparing the results with those of the image comparison measure. Unfortunately this is extremely difficult.

An extensive amount of work relating to image manipulation routines has been published. Often these articles compare routines in terms of the quality of images produced. However, it is impossible to draw any parallels between two similar routines without a reliable measure of image quality. This is evident in the design of compression algorithms, for which the need to measure the quality of a reconstructed image is essential in determining the compression algorithm's performance. In fact many compression algorithms' have their parameters set to minimise RMS error [7].

The following presents a new approach to image comparison employing phase information. The importance of phase information in an image is highlighted by an experiment performed by Oppenheim and Lim [6] involving constructing a synthetic image from two seperate images, by extracting the phase information from one and the magnitude information from the other. The resulting image clearly corresponds to the image from which the phase information was taken [6]. From this result it is evident that phase is important in an image, and consequently a measure based on phase may be used to overcome some of the flaws inherent in RMS type error measures.

The new phase based metric is a measure of perceptual distortion [3], in that it produces a measure of relative difference. This result is intuitive as the distortion measure is based on a comparison of the phase between two images, not on a compression ratio or simple tally of pixel difference. Consequently, differences can be weighted. For example, a large translation is not measured as a large distortion. Thus rectifying another of the problems inherent to the RMS.

The new measure presented in this paper calculates a *phase difference* between frequency information extracted from each image. The phase difference is then used to calculate a measure of distortion and an estimate of phase displacement. The estimated phase displacement is used to calculate a measure of translation. A displacement is measured by observing the offset between phase at each point. A constant offset greater than zero implies a translation between images. Distortion is measured by calculating the standard and absolute deviations of the phase difference. Distortion can be calculated irrespective of a translation between images, since the phase difference between identical images is constant; the standard deviation will be zero, or near zero, with or without a displacement.

2. Frequency Analysis

Phase data are extracted from an image via frequency analysis. A common approach to frequency analysis is the *Fourier transform* [5]. Although providing the frequencies present in a signal, it unfortunately does not tell us where each frequency occurs, and consequently it is impossible to use the result of the Fourier transform to compare two distinct points in two separate images.

Based on work by Morlet et. al [4], the extraction of phase and amplitude information can be performed using wavelets in quadrature. A wavelet approach is employed as it allows the extraction of phase localised in both frequency and space.

Wavelet analysis involves creating a bank of filters in quadrature. The approach taken here, based on work by Kovesi [1, 2], employs a bank of geometrically scaled *Log Gabor* wavelets. These filters have a Gaussian transfer function when viewed on a logarithmic scale. Log Gabor filters are employed because they offer a good compromise between *spatial localisation* and *frequency localisation.* Log Gabor filters are also preferred over other wavelets since they allow arbitrarily large bandwidth filters to be constructed while still maintaining a zero DC component in the even-symmetric filter. A zero DC component cannot be maintained in Gabor functions over one *octave*.

The frequency a filter targets depends on the frequency of the sine/cosine wave and the bandwidth/localisation in space is controlled by the width of the Gaussian envelope. Using filters in quadrature enables the calculation of amplitude and phase data for a particular scale/frequency at a given spatial location.

Each pair of even and odd symmetric filters is created by geometrically rescaling the respective original wavelet. The bank of filters is then convolved with the image to generate frequency information.

2.1. Calculating Phase and Amplitude Information

Convolution of a signal with even and odd complex valued Log Gabor filters results in an array of complex valued numbers, at each point in the signal, of the form $a + ib$. We can think of a as the result of convolution between the image and even filter, and b as the result of convolution between the image and odd filter. From this result localised phase information θ_n, for a filter of scale n, is calculated by

$$\theta_n = \text{atan2}(b, a), \tag{1}$$

and localised amplitude information A_n is calculated by

$$A_n = \sqrt{a^2 + b^2}. \tag{2}$$

Simply put, wavelet frequency analysis decomposes a signal into a series of basis functions representing each frequency recorded. At each point we have phase and amplitude pairs, for each scale of filter.

2.2. Calculating Phase Angle Difference

For purposes of comparison, a *difference* must be calculated between the phase angle data. This difference must account for the wrap around problem. The wrap around problem is caused by the fact that the difference between $359°$ and $1°$ is $2°$, not $358°$ as calculated by a simple subtraction. Hence, given a phase angle θ_1, and phase angle θ_2, a *phase difference* $\Delta\theta$ can be calculated based on the sine and cosine difference of angle laws, as

$$\begin{aligned}
\cos(\Delta\theta) &= \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2), \\
\sin(\Delta\theta) &= \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1), \\
\Delta\theta &= \text{atan2}(\sin(\Delta\theta), \cos(\Delta\theta)).
\end{aligned}$$

The resulting phase difference can now be used to determine the displacement or distortion as required.

3. Displacement

By converting the phase difference at different scales to an equivalent displacement it is possible to relate the phase differences at different scales to a useful measure of image translation. Given a phase difference $\Delta\theta$ at a point, an estimate of the translation between images at that point is given by the phase displacement $\Delta\Phi$, and is calculated by

$$\Delta\Phi = \frac{\Delta\theta}{(2\pi)} \times \lambda_s, \qquad (3)$$

where λ_s is the wavelength of the filter at scale s.

A displacement is calculated for each point in the signal, for each scale of filter. If the two signals being compared are identical, bar translation, then the measured displacement should be consistent across all points.

4. Distortion

Calculating the distortion of an image using phase information is based on the premise that the localised estimated displacement at all scales between two identical images will be constant, regardless of translation. Moreover, if a standard deviation is calculated over the displacement estimates, the deviation from the mean only increases if there are actual differences between images. Based on this assumption, two methods are proposed for the estimation of distortion: a standard deviation and an absolute deviation [1]. An absolute deviation is implemented along with a standard deviation since taking the absolute value of difference rather than the square root of a squared difference can be more sensitive to minor changes. This increased sensitivity is desirable when measuring slight distortions.

The standard deviation σ, and the absolute deviation α, are calculated as follows:

$$\sigma = \sqrt{\sum_i^N \frac{(\Delta\Phi_i - \overline{\Delta\Phi})^2}{N}}, \qquad (4)$$

$$\alpha = \sum_i^N \frac{|(\Delta\Phi_i - \overline{\Delta\Phi})|}{N}, \qquad (5)$$

where N is the number of pixels in the image, $\Delta\Phi_i$ is the phase displacement calculated at pixel i, and $\overline{\Delta\Phi}$ is the mean phase displacement value.

4.1. Distortion Results

This section presents results for both the new phase based measure and the RMS error. The results presented consist of an original reference image and the reference image compressed and reconstructed. This is done for increasing levels of compression, and translations of zero and six pixels between images. Figures 1, 2, and 3 display the original, 55% compressed, and 95% compressed images respectively. Compression was perfromed with the JPEG lossy image compression standard.

Figure 1. Original test image.

Figure 2. Test image compressed to 55%.

Figure 4 presents the measured distortion as calculated for the new phase based measure when applied to the image in Figure 1 and its respective compressed versions. This is a typical result for general images. Note that even for a translation of six pixels the effect of the translation on the measured distortion is minimal.

Figure 5 illustrates the results for the RMS error when applied to the same test image as in Figure 4. The effect of translation on the RMS is clearly evident. In particular, note that the RMS error decreases as compression increases when the image is translated. Note also the extreme increase in measured distortion for any translation between images. Figure 5 is typical of the results produced by the RMS error.

Figure 3. Test image compressed to 95%.

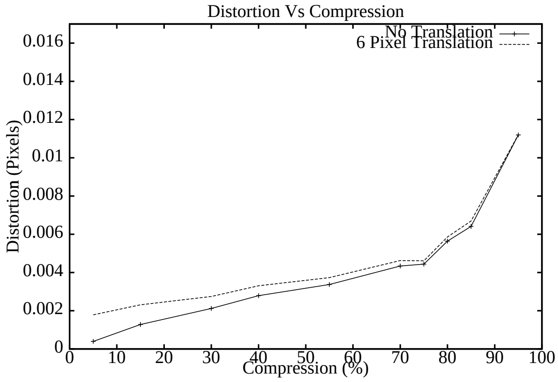

Figure 4. Measured distortion for the test image, with a displacement of 0 and 6 pixels respectively.

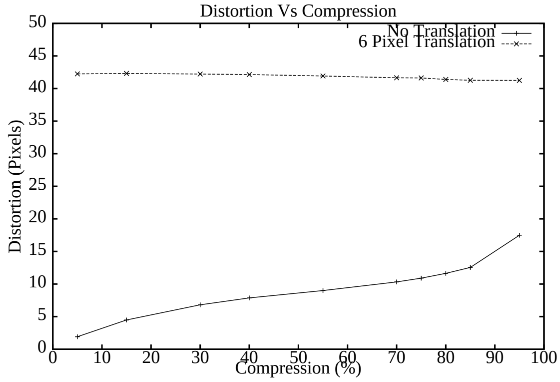

Figure 5. The distortion returned by the RMS error when applied to the test image for displacements of 0 and 6 pixels respectively.

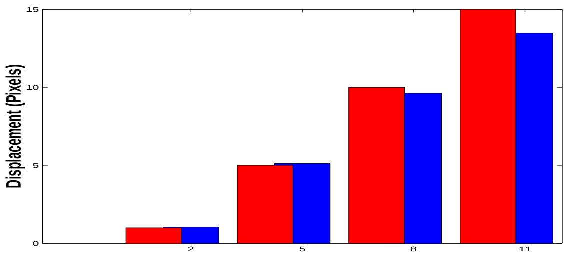

Figure 6. Measured displacement for the test image compared to itself. Actual displacement is indicated by the grey bar.

5. Translation Results

Beyond translations of 15 pixels, the phase based analysis technique is only useful as a relative metric: large translations cause measured distortions to increase considerably. However, as illustrated in Figure 6 and Figure 7, the estimate of the translation is accurate to two pixels. The grey bars in Figures 6 and 7 indicate actual translation. The black bars indicate measured translation. Note the accuracy decreases with each increase in translation.

6. Validation

The lack of a formal validation makes the task of assessing the performance of the new phase based measure extremely difficult. It was decided that a correct result was one such that an image of higher compression is given a higher measured distortion. Similarly an image that is modified from the original is also deemed to have been distorted. The criteria for performance rating has the perhaps undesirable effect that a sharpened image, deblurred image, or any similarly enhanced image, is categorized as distorted. Given that the new measure is a relative one, this result is of little importance, and an image, even if appearing better than the original, is still distorted relative to the original. The important factor is whether the image has been distorted, and type of distortion, not whether the image is more aesthetically pleasing.

The proposed measure is a relative metric and it can not be used as an absolute number. However, as indicated by the problems with the RMS and SNR measures, image quality cannot be judged on a single number. Rather, a range of image attributes must be considered [3].

7. Conclusion

Comparing images is a highly subjective task, and one that is especially hard to show correct. A new image comparison metric based on using frequency information has

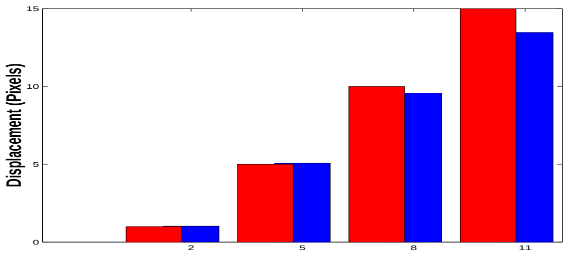

Figure 7. Measured displacement for the test image compared with the 85% compressed version. Actual displacement is indicated by the grey bar.

been presented and shown to alleviate some of the problems inherent with the RMS and SNR errors.

Image quality is extremely difficult to measure. Moreover, to date there is no satisfactory measure, or definition, of image quality. Thus making the task of validating the phase based measure extremely difficult.

The new measure presented is based on phase information and this is clearly important to the human visual system. In particular, the new measure is able to separate translation from distortion. However, more work is required in understanding the effect of the filter shape on the reliability of frequency information extracted.

References

[1] P. Kovesi. *Invariant Measures of Image Features From Phase Information*. PhD thesis, The University of Western Australia, 1996.

[2] P. Kovesi. Symmetry and asymmetry from local phase. In *Tenth Australian Joint Conference on Artificial Intelligence*. AI'97, December 1997.

[3] G. P. Lorenzetto. A new approach to image comparison via frequency analysis. Honours thesis, Department of Computer Science, The University of Western Australia, 1998.

[4] J. Morlet, G. Arens, E. Fourgeau, and D. Giard. Wave propagation and sampling theory - part II: Sampling theory and complex waves. *Geophysics*, 47:222–236, February 1982.

[5] N. Morrison. *Introduction to Fourier Analysis*. John Wiley & Sons, Brisbane, 1994.

[6] A. V. Oppenheim and J. S. Lim. The importance of phase in signals. In *Proceedings of the IEEE*, volume 69, pages 529–541, May 1981.

[7] D. L. Wilson, A. J. Baddeley, and R. A. Owens. A new metric for grey-scale image comparison. *International Journal of Computer Vision*, 24(1):5–17, 1997.